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Real-Time Determination of Absolute Frequency in Continuous-Wave Terahertz Radiation with a Photocarrier Terahertz Frequency Comb Induced by an Unstabilized Femtosecond Laser

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Abstract A practical method for the absolute frequency measurement of continuous-wave terahertz (CW-THz) radiation uses a photocarrier terahertz frequency comb (PC-THz comb) because of its ability to realize real-time, precise measurement without the need for cryogenic cooling. However, the requirement for precise stabilization of the repetition frequency (f_{rep}) and/or use of dual femtosecond lasers hinders its practical use. In this article, based on the fact that an equal interval between PC-THz comb modes is always maintained regardless of the fluctuation in f_{rep}, the PC-THz comb induced by an unstabilized laser was used to determine the absolute frequency f_{THz} of CW-THz radiation. Using an f_{rep}-free-running PC-THz comb, the f_{THz} of the frequency-fixed or frequency-fluctuated active frequency multiplier chain CW-THz source was determined at a measurement rate of 10 Hz with a relative accuracy of 8.2×10^{-13} and a relative precision of 8.8×10^{-12} to a rubidium frequency standard. Furthermore, f_{THz} was correctly determined even when fluctuating over a range of 20 GHz. The proposed method enables the use of any commercial femtosecond laser for the absolute frequency measurement of CW-THz radiation.

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1 Introduction

Along with increasing demand for higher speed wireless communication, terahertz (THz) radiation (frequency = 0.1 to 10 THz) has attracted attention as a new band for wireless communications [1, 2]. Recent progress in the development of continuous-wave THz (CW-THz) sources, such as the THz quantum cascade laser (THz-QCL) [3], the uni-traveling-carrier photodiode (UTC-PD) [4], or the resonant tunnel diode (RTD) [5], have boosted research in THz wireless communications. When the THz radiation is used for wireless communication, its frequency allocation is important to secure necessary and sufficient bandwidth without interference with other applications. However, the allocation of frequencies higher than 0.275 THz has not yet been established. If precise frequency allocation of the THz band is established in the same manner as other radio waves, it will be necessary to determine the absolute frequency of various types of CW-THz sources correctly in real time. However, conventional methods of measuring the absolute frequency of CW-THz radiation, such as electrical heterodyne methods [6, 7] or optical interferometric methods, need cryogenic cooling of the mixer or detector to suppress thermal noise. For practical and widespread use, a real-time method of measuring the absolute frequency without the need for cryogenic cooling is strongly required.

One possible method that meets this requirement involves the use of a photocarrier THz frequency comb (PC-THz comb) [8–10]. When femtosecond mode-locked laser light is incident onto a photoconductive antenna (PCA) without an electrical bias, namely a PCA for detecting THz radiation, a PC-THz comb is generated in the PCA. Due to a coherent photoconductive process in the PCA, the PC-THz comb is a harmonic frequency comb of the laser repetition rate (f_{rep}) without a carrier-envelope-offset frequency (f_{ceo}), namely, an f_{ceo}-free comb. If f_{rep} is phase-locked to a frequency standard in microwave or radio-frequency (RF) regions by laser control, the PC-THz comb can be used as a precise ruler for measuring THz frequency. To be more specific, when measured CW-THz radiation is incident on a PCA having a PC-THz comb, the PC-THz comb acts as a local oscillator with known multiple frequencies covering the THz region. Therefore, the absolute frequency f_{THz} of the CW-THz radiation can be determined from f_{rep}, the PC-THz comb mode number m nearest in frequency to the CW-THz radiation, and the beat frequency f_{beat} generated by the photoconductive mixing between the PC-THz comb mode and the CW-THz radiation [11–15].

The photoconductive mixing of CW-THz radiation with a PC-THz comb has been successfully applied to the absolute frequency measurement of narrow-linewidth CW-THz sources [11–15] and even multiple CW modes in broadband THz comb [12, 16–19] at room temperature. When a single, f_{rep}-stabilized PC-THz comb is used for the absolute frequency determination of CW-THz radiation, two f_{beat} signals have to be measured before and after shifting f_{rep} of the PC-THz comb by the laser control in order to determine m and sign of f_{beat} [11–13]. Such time-sequential, two-step measurement with a single, f_{rep}-stabilized PC-THz comb has been an obstacle in applying this technique to the real-time absolute frequency measurement of frequency-fluctuating CW-THz radiation. More importantly, the need for a precisely stabilized femtosecond laser often makes this frequency measurement less practical.

An unstabilized femtosecond laser has been also used for high-precision frequency measurements of CW-THz radiation together with a simple and flexible algorithm of instantaneous frequency calculation [13]. However, use of expensive and bulky mode-locked Ti:Sapphire laser and the associated maintenance limit the practical use of this method. Furthermore, there have been no attempts to perform the real-time determination of f_{THz} because of time-sequential, two-step measurement. Recently, to achieve real-time absolute frequency measurement with unstabilized femtosecond laser, a pair of unstabilized PC-THz combs with different f_{rep} values (f_{rep1} and f_{rep2}), namely unstabilized dual PC-THz combs, has been effectively used for real-time absolute frequency measurement of CW-THz radiation with rapid, large frequency variations [15]. This is based on the fact that a PC-THz comb always functions as a frequency ruler with an equal interval and a linear scale at every moment regardless of whether or not f_{rep} stabilization is used. Only if the f_{rep} is correctly monitored in real time, the PC-THz comb can be still used as a precise ruler for measuring THz frequency. Absolute frequency measurement with a precision of 4.0×10^{-11} was achieved at a measurement rate of 100 Hz by simultaneous measurement of two f_{rep} signals (f_{rep1} and f_{rep2}) and two f_{beat} signals (f_{beat1} and f_{beat2}) for the dual unstabilized PC-THz combs. However, the need for dual femtosecond lasers is the last obstacle for the practical use of this method, even though unstabilized lasers can be used.

In the work described in this article, we demonstrated real-time, absolute frequency measurement based on a single PC-THz comb induced by a compact, turnkey, mode-locked fiber laser without frequency stabilization. The absolute frequency of frequency-fluctuating CW-THz radiation was determined by rapid consecutive measurement of f_{rep} and f_{beat} in an f_{rep}-modulated or f_{rep}-free-running PC-THz comb.

2 Principle of Operation

2.1 Use of f_{rep}-Modulated PC-THz Comb

When femtosecond laser light with repetition frequency f_{rep} is incident on a PCA for detecting THz radiation, a PC-THz comb is induced in PCA. Then, when CW-THz radiation with frequency f_{THz} to be measured is incident on the PCA, a group of beat signals are generated as a current signal from the PCA by photoconductive mixing between the PC-THz comb modes and the CW-THz radiation. The photoconductive mixing between the CW-THz radiation and the PC-THz comb mode m nearest in frequency to the CW-THz radiation generates a beat signal at the lowest frequency, namely, f_{beat}. Therefore, f_{THz} is given by

$$f_{\mathrm{THz}} = m f_{\mathrm{rep}} \pm f_{\mathrm{beat}} \tag{1}$$

Since f_{rep} and f_{beat} can be measured directly in the RF region, the value of m and the sign of f_{beat} have to be determined to obtain f_{THz}.

In previous research, dual unstabilized PC-THz combs with different frequency spacing were used for simultaneous measurement of f_{rep1}, f_{rep2}, f_{beat1}, and f_{beat2}, and the value of m and the sign of f_{beat} were determined in real time based on them [15]. However, use of dual PC-THz combs is not always essential if the measurement rate of f_{rep} and f_{beat} is much faster than the temporal fluctuation of f_{THz}. This is because two consecutive measurements of f_{rep} and f_{beat} with a single PC-THz comb can be regarded as being equivalent to the simultaneous measurement of f_{rep1}, f_{rep2}, f_{beat1}, and f_{beat2} with dual PC-THz combs. Therefore, a single, unstabilized PC-THz comb can be used for the real-time, absolute frequency measurement of frequency-fluctuating CW-THz radiation.

We next explain how to determine the value of m and the sign of f_{beat} with a single, unstabilized PC-THz comb in real time. Figure 1 shows the principle of determining f_{THz} in real time when the frequency spacing in a single PC-THz comb is modulated, namely, an f_{rep}-modulated PC-THz comb, where (a) shows the spectral behavior of the f_{rep}-modulated PC-THz comb and the measured CW-THz radiation (freq. $=f_{THz}$) and (b) shows the corresponding description of f_{rep} and f_{beat} in the time domain. When f_{rep} is sinusoidally modulated between f_{rep_min} and f_{rep_max} at a modulation frequency f_{mod}, f_{beat} is also sinusoidally modulated between f_{beat_min} and f_{beat_max} at f_{mod} in synchronization with f_{rep}. If f_{mod} is much faster than the temporal fluctuation of f_{THz}, then f_{rep_min}, f_{rep_max}, f_{beat_min}, and f_{beat_max} can be regard as f_{rep1}, f_{rep2}, f_{beat1}, and f_{beat2} in dual unstabilized PC-THz combs [15], respectively. Therefore, the m value can be obtained in real time by

$$m = \frac{f_{beat\_max} - f_{beat\_min}}{f_{rep\_max} - f_{rep\_min}} \tag{2}$$

The sign of f_{beat} in Eq. (1) can be determined by a phase relation between modulated waveforms of f_{rep} and f_{beat}. On one hand, if the sinusoidal wave of f_{beat} is out of phase with that of f_{rep}, the sign is positive since $f_{THz} > mf_{rep\_max}$ [see Fig. 1a, b]. On the other hand, if the sinusoidal wave of f_{beat} is in phase with that of f_{rep}, the sign is negative since $f_{THz} < mf_{rep\_min}$ (not shown). Therefore, f_{THz} can be obtained by

$$\begin{array}{ll} f_{THz} == \dfrac{f_{beat\_max} - f_{beat\_min}}{f_{rep\_max} - f_{rep\_min}} f_{rep\_mean} + f_{beat\_mean} & \left\langle f_{rep} \text{ (out-of-phase) } f_{beat} \right\rangle \\ f_{THz} = \dfrac{f_{beat\_max} - f_{beat\_min}}{f_{rep\_max} - f_{rep\_min}} f_{rep\_mean} - f_{beat\_mean} & \left\langle f_{rep} \text{ (in-phase) } f_{beat} \right\rangle, \end{array} \tag{3}$$

where f_{rep_mean} and f_{beat_mean} are mean values for modulated f_{rep} and f_{beat}. Finally, f_{THz} can be determined by measuring f_{rep_max}, f_{rep_min}, f_{rep_mean}, f_{beat_max}, f_{beat_min}, and f_{beat_mean}.

2.2 Use of f_{rep}-Free-Running PC-THz Comb

One may consider that the need for external modulation of the cavity length to modulate f_{rep} is still an obstacle for the widespread use of this method, even though laser stabilization is

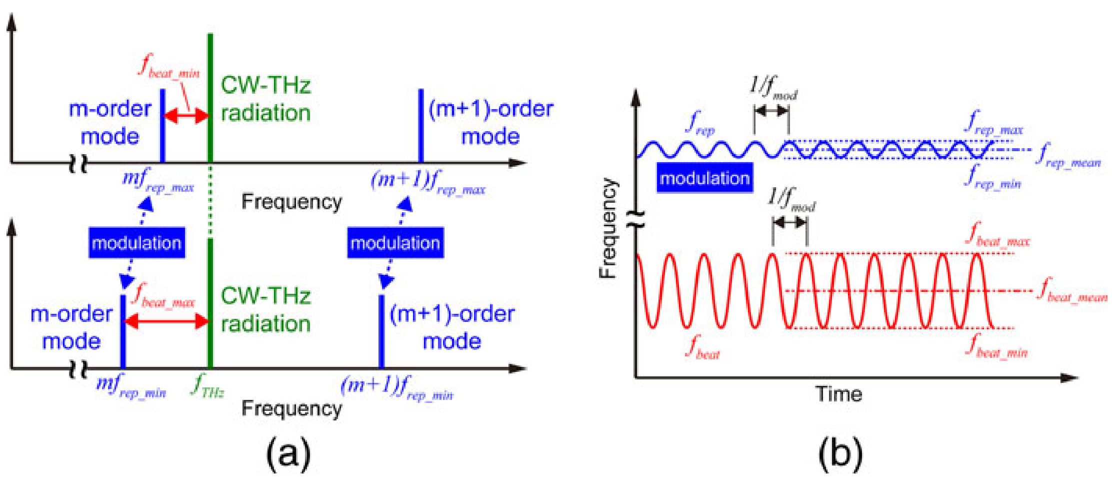

Fig. 1 Principle of real-time measurement with f_{rep}-modulated PC-THz comb. **a** Spectral behavior of f_{rep}-modulated PC-THz comb and measured CW-THz radiation and **b** the corresponding description of f_{rep} and f_{beat} in the time domain

unnecessary. In an unstabilized PC-THz comb without external modulation, the frequency spacing f_{rep} varies every moment due to external disturbances to the laser cavity length, such as temperature changes, mechanical vibrations, acoustic vibrations, and so on. If such an f_{rep}-free-running PC-THz comb is used for the photoconductive mixing with CW-THz radiation at f_{THz} to be measured, the resulting f_{beat} signal also varies every moment reflecting the change of f_{rep}, as shown in Fig. 2a, b. Such synchronized changes of f_{rep} and f_{beat} can also be used for the real-time determination of the value of m and the sign of f_{beat} if the temporal changes of f_{rep} and f_{beat} are faster than the temporal fluctuation of f_{THz}. When we obtain f_{rep1}, f_{rep2}, f_{beat1}, and f_{beat2} by two consecutive measurements of f_{rep} and f_{beat}, m can be obtained by

$$m = \frac{|f_{beat2} - f_{beat1}|}{|f_{rep2} - f_{rep1}|} \tag{4}$$

Finally, f_{THz} can be determined by

$$\begin{aligned} f_{THz} &= \frac{|f_{beat2} - f_{beat1}|}{|f_{rep2} - f_{rep1}|} f_{rep1} + f_{beat1} \quad \left\langle \frac{f_{beat2} - f_{beat1}}{f_{rep2} - f_{rep1}} < 0 \right\rangle \\ f_{THz} &= \frac{|f_{beat2} - f_{beat1}|}{|f_{rep2} - f_{rep1}|} f_{rep1} - f_{beat1} \quad \left\langle \frac{f_{beat2} - f_{beat1}}{f_{rep2} - f_{rep1}} > 0 \right\rangle \end{aligned} \tag{5}$$

3 Experimental Setup

3.1 Use of f_{rep}-Modulated PC-THz Comb

Figure 3 shows a schematic diagram of the experimental setup for absolute frequency measurement with an f_{rep}-modulated PC-THz comb. The setup included an f_{rep}-modulated femtosecond laser, a PCA for THz detection, and data acquisition electronics. We used a mode-locked Er-doped fiber laser (C-COMB and P100, Menlo Systems; center wavelength = 1550 nm, pulse duration = 56 fs, $f_{rep} \approx 100$ MHz) to generate a PC-THz comb in the PCA. The repetition frequency f_{rep} was not

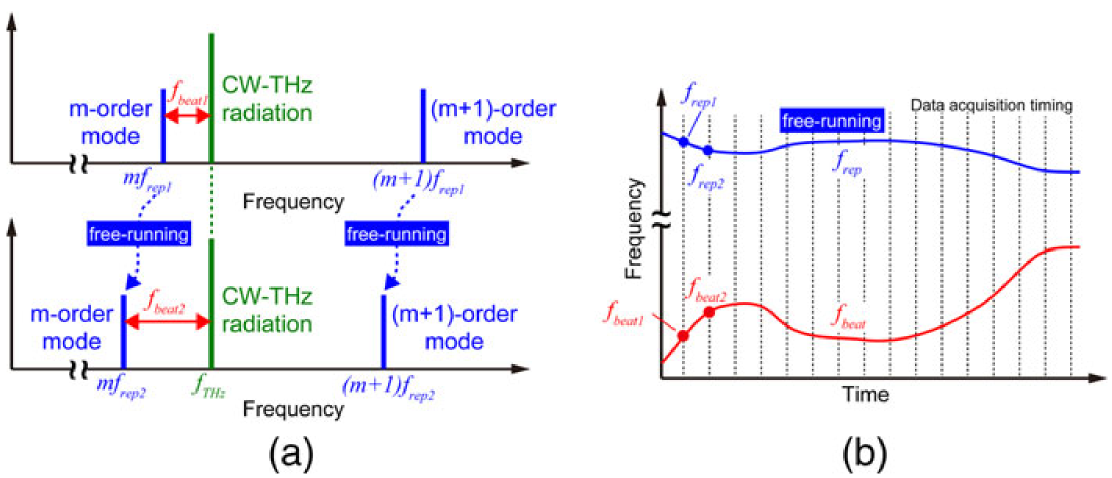

Fig. 2 Principle of real-time measurement with f_{rep}-free-running PC-THz comb. **a** Spectral behavior of f_{rep}-free-running PC-THz comb and a measured CW-THz radiation and **b** the corresponding description of f_{rep} and f_{beat} in the time domain

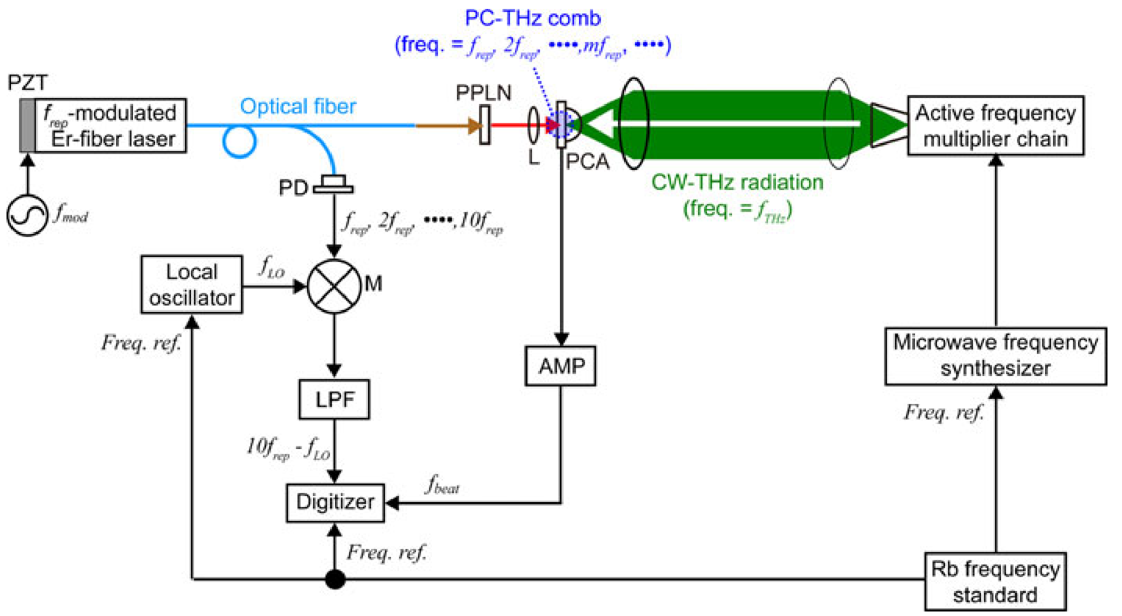

Fig. 3 Experimental setup for absolute frequency measurement with f_{rep}-modulated PC-THz comb. *PPLN* periodically-poled-lithium-niobate crystal, *L* objective lens, *PCA* photoconductive antenna, *PC-THz-comb* photocarrier THz frequency comb, *M* mixer, *LPF* low-pass filter, *AMP* current preamplifier

stabilized and could be modulated within a frequency deviation of ±50 Hz by a piezoelectric actuator (PZT; cut-off freq. ≈ 100 Hz) to change the cavity length. A portion of the laser light was detected with a fast photodetector (PD). The 10-th harmonic component of f_{rep}, namely $10f_{rep}$, was electrically mixed with an output signal (freq. = f_{LO} = 980 MHz) from a local oscillator, and a beat signal between them, namely $10f_{rep}-f_{LO}$ (≈20 MHz), was extracted by a low-pass filter (LPF).

The output light from the fiber laser was converted into second-harmonic-generation (SHG) light with a periodically-poled-lithium-niobate (PPLN) crystal. The resulting 775 nm light was incident on a bowtie-shaped, low-temperature-grown GaAs PCA without an electrical bias, resulting in the generation of a PC-THz comb in the PCA. In this experiment, we measured CW-THz radiation (frequency f_{THz}) from an active frequency multiplier chain (Millitech AMC-10-R0000, multiplication factor = 6, tuning range = 75–110 GHz, linewidth <0.6 Hz, and average power = 2.5 mW), which multiplied the output frequency of a microwave frequency synthesizer (Agilent E8257D, frequency = 12.5–18.33 GHz, and linewidth <0.1 Hz) by six. The frequency synthesizer was phase-locked to a rubidium frequency standard (Stanford Research Systems FS725 with frequency = 10 MHz, accuracy = 5×10^{-11}, stability = 2×10^{-11} at 1 s). When the CW-THz radiation emitted from this source was incident on the PCA together with the 775 nm light, a group of beat signals between the CW-THz radiation and the PC-THz comb modes was generated as an output current in the PCA by photoconductive mixing between them. The f_{beat} signal was amplified by a current preamplifier (AMP; bandwidth = 40 MHz and transimpedance gain = 100,000 V/A). The temporal waveforms for $10f_{rep}-f_{LO}$ and f_{beat} were acquired simultaneously by a fast digitizer (resolution = 14 bit, sampling rate = 100 MHz), which used the output signal from the frequency standard for an external clock in the digitizer. From the acquired temporal waveforms, we obtained instantaneous values of f_{rep} and f_{beat} by the instantaneous-frequency-calculation algorithm involving a Fourier transform (data length = 1,000,000), digital frequency filtering (bandpass width = 10 kHz), an inverse Fourier transform, a Hilbert transform, the time differential of the instantaneous phase, and signal averaging [13]. Finally, we determined f_{THz} by substituting f_{rep_max}, f_{rep_min}, f_{rep_mean}, f_{beat_max}, f_{beat_min}, and f_{beat_mean} into Eq. (3).

3.2 Use of f_{rep}-Free-Running PC-THz Comb

Figure 4 shows a schematic diagram of the experimental setup for the absolute frequency measurement with an f_{rep}-free-running PC-THz comb, which is similar to that shown in Fig. 3. Only the differences between them are described here. SHG light of another mode-locked Er-doped fiber laser (Femtolite AS-20-STD, IMRA; center wavelength = 780 nm, pulse duration = 100 fs, $f_{rep} \approx 50.827$ MHz) was used to generate an f_{rep}-free-running PC-THz comb in the PCA. The 15-th harmonic component of f_{rep}, namely $15f_{rep}$, was electrically mixed with an output signal (freq. = f_{LO} = 763,388,000 Hz) from a local oscillator, and a beat signal between them, namely $15f_{rep}-f_{LO}$ (≈1 MHz), was measured. The f_{beat} signal was amplified by a current preamplifier (AMP; bandwidth = 10 MHz and transimpedance gain = 100,000 V/A). The temporal waveforms for $15f_{rep}-f_{LO}$ and f_{beat} were acquired simultaneously by a digitizer (resolution = 14 bit, sampling rate = 10 MHz). Finally, we determined f_{THz} by substituting the instantaneous values of f_{rep} and f_{beat} into Eq. (5).

4 Results

4.1 Use of f_{rep}-Modulated PC-THz Comb

First, we measured the absolute frequency of fixed-frequency CW-THz radiation (f_{THz} = 100,001,004,000 Hz) by using the f_{rep}-modulated PC-THz comb. To do so, f_{rep} was sinusoidally modulated within about 3 Hz at a modulation frequency of 100 Hz (= f_{mod}). From Eq. (2), uncertainty of m must be less than ±0.5. To avoid the incorrect determination of m, we have to determine $|f_{beat\_max}-f_{beat\_min}|$ within a frequency error of ±1.5 Hz because $|f_{rep\_max}-f_{rep\_min}|$ = 3 Hz. From a relation between measurement rate and frequency error in our previous research (sampling rate of the digitizer = 100 MHz) [15], the measurement rate of 10 Hz, or the measurement period of

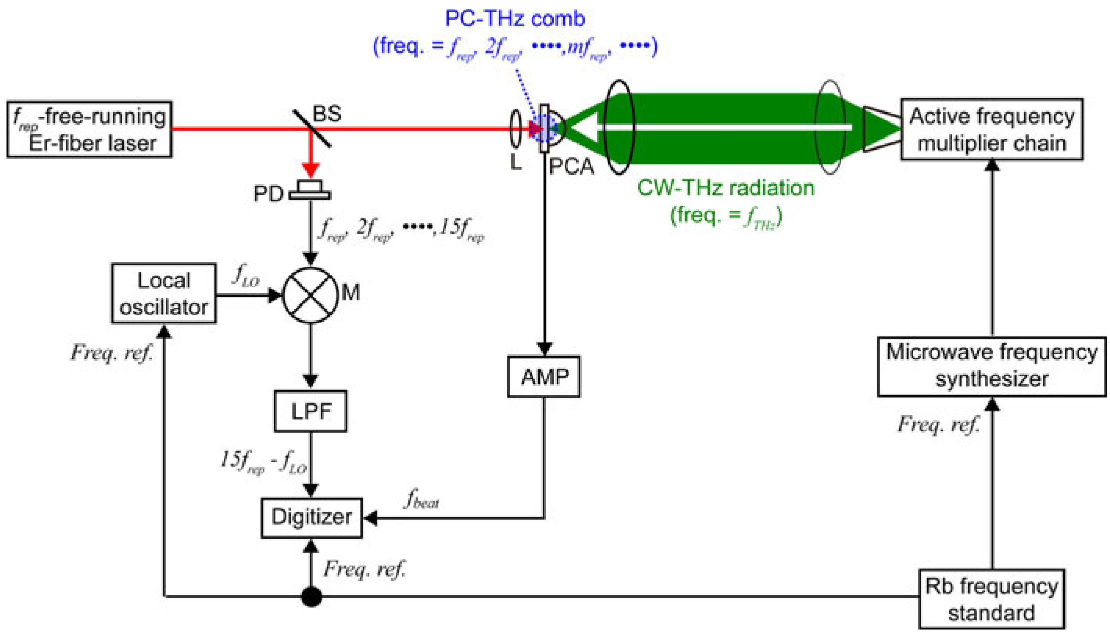

Fig. 4 Experimental setup for absolute frequency measurement with f_{rep}-free-running PC-THz comb. *BS* beam splitter, *L* objective lens, *PCA* photoconductive antenna, *PC-THz-comb* photocarrier THz frequency comb, *M* mixer, *LPF* low-pass filter, *AMP* current preamplifier

100 ms, is suitable to suppress the frequency error below 1 Hz. Figure 5a, b show temporal changes of the instantaneous values of f_{rep} and f_{beat} (acquisition time = 100 ms). We consider that the increased noise in f_{rep} after 90 ms was caused by errors in the instantaneous-frequency-calculation algorithm rather than the data acquisition error with the digitizer. We confirmed that both f_{rep} and f_{beat} were modulated at 100 Hz. The frequency deviation of f_{beat} should be m-times larger than that of f_{rep} from Eq. (2). Since (f_{rep_max}–f_{rep_min}) and (f_{beat_max}–f_{beat_min}) were respectively 3.1446 Hz and 3144.6 Hz, from curve fitting analysis with a sinusoidal wave for the temporal waveform during a period of 100 ms, the m value was determined to be 1000. On the other hand, the sign of f_{beat} was positive because f_{beat} was out-of-phase with f_{rep}. We further obtained mean values for modulated f_{rep} and f_{beat}, namely f_{rep_mean} and f_{beat_mean}, of 100,000,007.009 Hz and 996,520.005 Hz for 100 ms. In this way, we determined the f_{THz} value every 100 ms, or at a measurement rate of 10 Hz, as shown in Fig. 5c. For 100 repetitive measurements of f_{THz} at a measurement rate of 10 Hz, the mean and standard deviation of f_{THz} were 100,001,003,999.996 Hz and 1.5 Hz, respectively. These values were comparable with those in the absolute frequency measurement using the f_{rep}-stabilized PC-THz comb [15]. This result clearly indicated that the PC-THz comb correctly worked as a precise frequency ruler even when the femtosecond laser was unstabilized and modulated. The achieved accuracy and precision of the absolute frequency measurement were 4.0×10^{-14} and 1.5×10^{-11} at a measurement rate of 10 Hz, respectively. It should be noted that this accuracy and precision indicate the relative values to the frequency standard because the output signal of the frequency standard was used as a common time-base signal for the CW-THz source, the external clock in the digitizer, and the local oscillator. The absolute accuracy and precision of frequency measurement are determined by the absolute accuracy and stability of the frequency standard or the relative values, whichever is worse. Although the absolute accuracy and precision in the present system were limited by the rubidium frequency standard, use of another frequency standard with better uncertainty, such as cesium frequency standard or optical frequency standard, will further improve the absolute accuracy and precision of the frequency measurement up to the relative values.

Next, we determined the absolute frequency of the frequency-fluctuating CW-THz radiation to demonstrate the capability of monitoring mode-hopping or free-running behavior in CW-THz sources. To do so, f_{THz} was made to stepwise jump at intervals of about +200 MHz four times from 99,801,000,000 to 100,401,000,000 Hz. In this case, the CW-THz radiation crossed

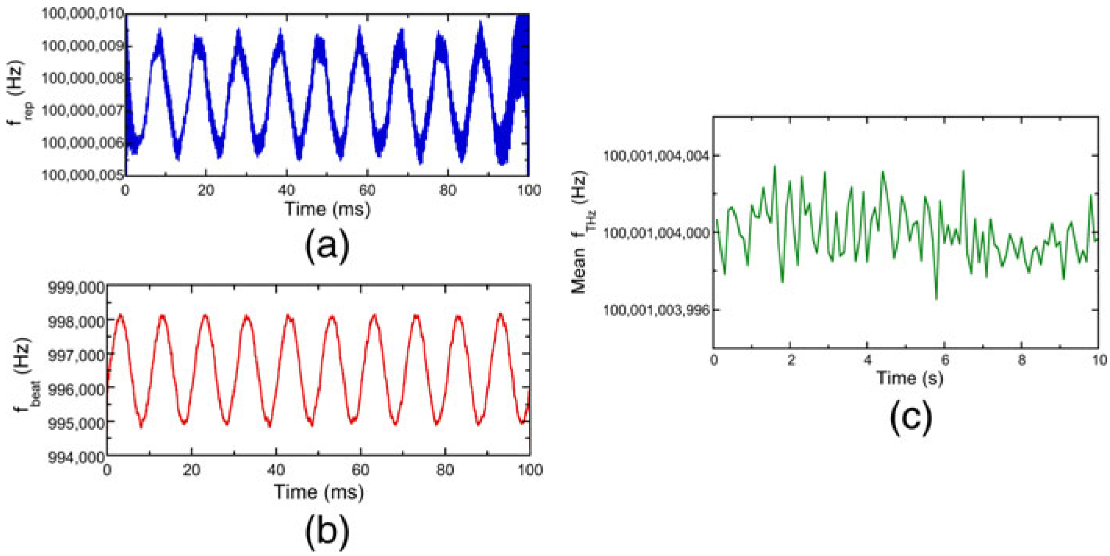

Fig. 5 Temporal changes of instantaneous values of **a** f_{rep} and **b** f_{beat} for measurement time of 100 ms. **c** Temporal changes of f_{THz} determined at a measurement rate of 10 Hz for a measurement time of 10 s

two PC-THz comb modes in every stepwise jump of f_{THz}. Figure 6a shows the temporal change of the value of m (blue plot) and the sign of f_{beat} (red plot) at a measurement rate of 10 Hz. Even though f_{THz} crossed the PC-THz comb modes suddenly, they could be determined without errors. Finally, we obtained the temporal change of f_{THz} from those values, as shown in Fig. 6b. The temporal behavior of f_{THz} reflected the stepwise tuning operation of the test source correctly. This demonstration reveals the potential of our method for measuring sudden fluctuations of f_{THz}.

4.2 Use of f_{rep}-Free-Running PC-THz Comb

In the previous section, we demonstrated absolute frequency measurement with an f_{rep}-modulated PC-THz comb. However, if femtosecond lasers without the need for stabilization and modulation of f_{rep} could be used in the proposed method, any commercial femtosecond laser could be used for the absolute frequency measurement of CW-THz radiation. To demonstrate the versatility of the proposed method, next we measured the absolute frequency in real time by use of an f_{rep}-free-running PC-THz comb induced by the unstabilized femtosecond laser.

It is important to measure $|f_{rep2}-f_{rep1}|$ and $|f_{beat2}-f_{beat1}|$ precisely to correctly determine m based on Eq. (4). When we use an f_{rep}-free-running PC-THz comb for absolute frequency measurement, the actual value of $|f_{rep2}-f_{rep1}|$ depends on the frequency fluctuation of unstabilized f_{rep}. On the other hand, the measurement precision of f_{rep} and f_{beat} with the instantaneous-frequency-calculation algorithm depends on the number of averaged signals, namely the measurement rate [15]. The measurement precision requirement of $|f_{beat2}-f_{beat1}|$ is less strict than that of $|f_{rep2}-f_{rep1}|$ because $|f_{beat2}-f_{beat1}|$ is the m-time multiple of $|f_{rep2}-f_{rep1}|$. Therefore, we have to consider the frequency fluctuation of unstabilized f_{rep} and the measurement precision of f_{rep} when the f_{rep}-free-running PC-THz comb is used.

We first investigated the frequency fluctuation of unstabilized f_{rep} by a RF frequency counter (Agilent 53220A) using the frequency standard as an external reference. Figure 7 shows the frequency fluctuation of f_{rep} with respect to the gate time of the counter immediately after switching on the femtosecond laser (0 min; red circle plot) and 35 min later (green triangle plot). Along with the elapsed time, the frequency fluctuation was reduced due to the thermal equilibrium of the laser cavity. On the other hand, to measure $|f_{rep2}-f_{rep1}|$ correctly, the measurement error of f_{rep} should be smaller than the fluctuation of unstabilized f_{rep}. A relation between the measurement error and the measurement rate in the instantaneous-frequency-calculation algorithm was given elsewhere; for

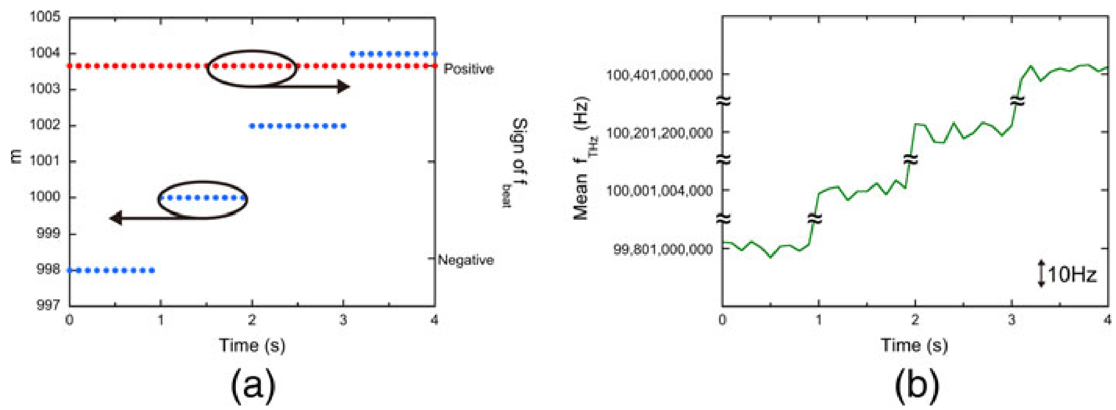

Fig. 6 Temporal changes of **a** the value of m and the sign of f_{beat} and **b** f_{THz} when f_{THz} was made to stepwise jump at intervals of +200 MHz four times from 99,801,000,000 Hz to 100,401,000,000 Hz during 4 s. The measurement rate was 10 Hz

example, the frequency error was 0.15 Hz at a measurement rate of 10 Hz [15]. From the comparison between this frequency error and the frequency fluctuation in Fig. 7, f_{rep} at 35 min after switching on the laser is too stable to determine the f_{rep} value without error at a measurement rate of 10 Hz. Therefore, we used PC-THz comb immediately after switching on, which is less stable than that at 35 min after.

We demonstrated the absolute frequency measurement of frequency-fixed CW-THz radiation (f_{THz} = 100,030,560,000 Hz) with the f_{rep}-free-running PC-THz comb. Figures 8a, b show temporal changes of f_{rep} and f_{beat}, in which the f_{rep} value drifted to lower frequency due to the thermal expansion of the cavity length, whereas the f_{beat} value shifted to higher frequency. Such opposite behavior of f_{rep} and f_{beat} indicated that the sign of f_{beat} was positive. On the other hand, the m value was determined to be 1968 during this measurement, based on Eq. (4). Finally, we determined the f_{THz} value at a measurement rate of 10 Hz, as shown in Fig. 8c. The mean and standard deviation of f_{THz} were 100,030,560,000.082 and 0.877 Hz in 100 repetitive measurements of f_{THz} at a measurement rate of 10 Hz. Therefore, the relative accuracy and precision of the absolute frequency measurement were 8.2×10^{-13} and 8.8×10^{-12}, respectively. These values were comparable with those in the absolute frequency measurement using the f_{rep}-modulated PC-THz comb, indicating that either an f_{rep}-modulated or f_{rep}-free-running PC-THz comb can be used for the precise frequency measurement of CW-THz radiation.

Finally, we demonstrated the absolute frequency measurement of frequency-jumping CW-THz radiation with the f_{rep}-free-running PC-THz comb. In this experiment, f_{THz} was made to stepwise jump at intervals of about +20 GHz, −500 MHz, and +5 GHz from 80,004,900,000 Hz during 10 s. The first, second, and third frequency jumps crossed +394 modes, −10 modes, and +99 modes in PC-THz comb, respectively. Figure 9a shows the temporal change of the m value (blue plot) and sign of f_{beat} (red plot), which correctly reflects the expected change of them. Figure 9b shows the temporal change of f_{THz}. Both large changes of several hundred MHz to a few tens of GHz and small changes of several Hz in f_{THz} [see an inset of Fig. 9b] were successfully measured together. This frequency measurement with a wide dynamic range was made possible by the use of a PC-THz comb with a high dynamic range of frequency. Most importantly, the demonstrated results indicated the possibility of using any commercial femtosecond laser for the precise frequency measurement of CW-THz radiation in real time.

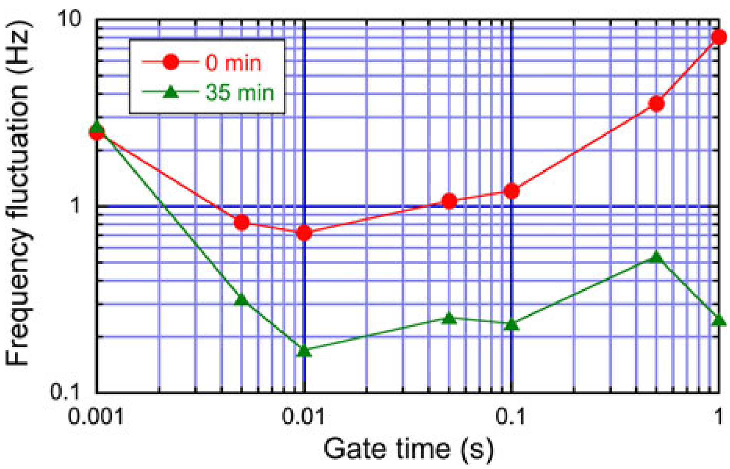

Fig. 7 Frequency fluctuation of unstabilized f_{rep} at different gate time immediately after switching on the femtosecond laser (0 min) and 35 min later

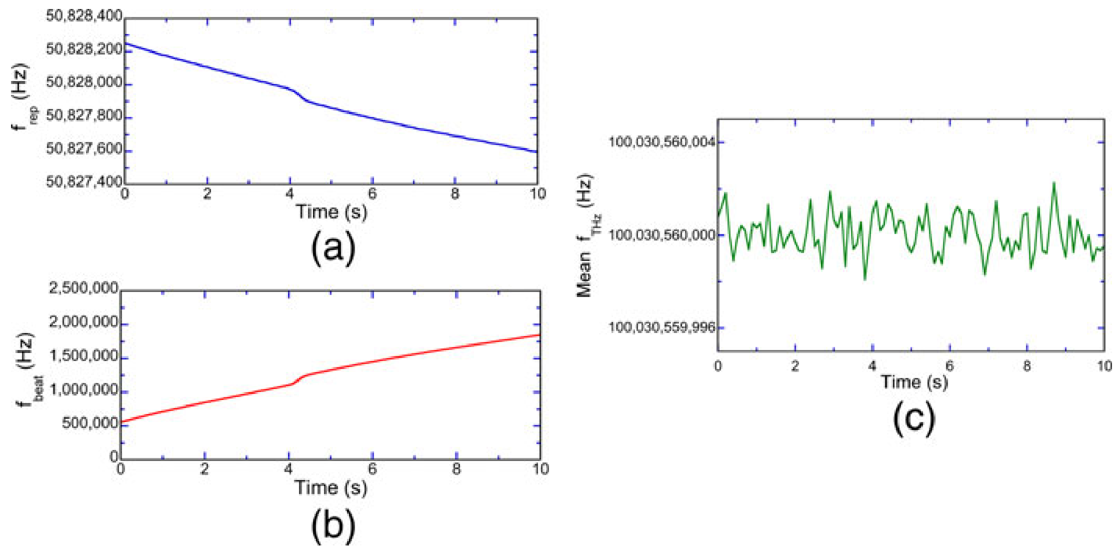

Fig. 8 **a** Temporal changes of instantaneous values of **a** f_{rep} and **b** f_{beat} for a measurement time of 10 s. **c** Temporal changes of f_{THz} determined at a measurement rate of 10 Hz for a measurement time of 10 s

5 Discussions

We first discuss error factors of m in the proposed method. Equations (2) and (4) hold true when two f_{beat} signals ($f_{\mathrm{beat\_max}}$ and $f_{\mathrm{beat\_min}}$, or, f_{beat1} and f_{beat2}) are generated by the same mode in PC-THz comb and signs of them are the same. On the other hand, the two f_{beat} signals, generated by different modes in PC-THz comb or whose signs are different to each other, cannot satisfy these equations. For largely fluctuating CW-THz CW-THz radiation, error of m may occur in a moment that CW-THz radiation suddenly crosses the PC-THz comb mode. For example, the lack of data plots in Fig. 9a was due to such error of m, indicating incorrect m value outside of the range of the vertical scale. If the CW-THz radiation always fluctuates across PC-THz comb modes or the spectrum of CW-THz radiation superimposes multiple PC-THz comb modes, the proposed method cannot be applied. Another factor to hinder the correct determination of m is the frequency measurement precision of f_{rep} and f_{beat} signals because uncertainty of m must be less than ±0.5 in Eqs. (2) and (4). Since the measurement precision with the instantaneous-frequency-calculation algorithm depends on the measurement rate [15], we have to select the suitable measurement rate and/or modulation deviation of f_{rep} by considering it. Also, f_{beat} signals around DC and $f_{\mathrm{rep}}/2$ may lead to errors because

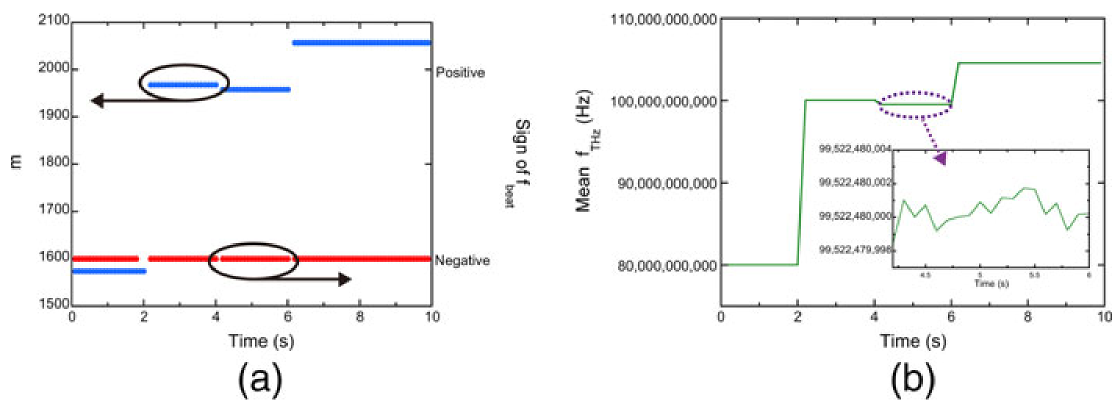

Fig. 9 Temporal changes of **a** the value of m and the sign of f_{beat} and **b** f_{THz} when f_{THz} was made to stepwise jump at intervals of +20 GHz, −500 MHz, and +5 GHz from 80,004,900,000 Hz during 10 s. The measurement rate was 10 Hz

of discontinuity of signal or coexistence of a mirror signal f_{rep}–f_{beat}. To avoid the coexistence of the mirror signal, the current preamplifier functions as a low-pass filter in addition to a transimpedance amplifier although the measurable range of f_{beat} is limited by the frequency bandwidth of it.

We next discuss a possibility to determine f_{THz} for rapidly fluctuating CW-THz radiation. In the real-time determination of f_{THz} with f_{rep}-modulated PC-THz comb, we assumed that the temporal fluctuation of f_{THz} should be much slower than f_{mod}. In the demonstration of Figs. 5 and 6, the measurement rate was 10 Hz because f_{rep} is modulated at 100 Hz by PZT. For faster modulation of f_{rep} over kHz, an electro-optic modulator can be used in the fiber laser cavity in place of PZT [20]. Also, optical frequency comb generators based on Mach-Zehnder modulator [21] or intracavity modulator [22] will be useful for this purpose. For more rapidly fluctuating CW-THz radiation, the real-time monitoring of the spectral shape may be important rather than the real-time determination of f_{THz}. In this case, an RF spectrum analyzer will be useful for real-time monitoring of the spectral shape of f_{THz} because the linewidth of PC-THz comb mode (typically, below 1 Hz) [11, 12] is much narrower than that of the fluctuating CW-THz radiation and hence the spectral shape of f_{beat} correctly reflects that of f_{THz}.

We finally discuss the measurable range of the proposed absolute frequency measurement. In this article, we demonstrated the real-time determination of f_{THz} using the CW-THz source around 0.1 THz. One may consider the availability of the proposed method above 0.275 THz because the high speed wireless communication will be allocated at higher frequency than 0.275 THz. In principle, the measurable range is limited by the spectral sensitivity of PCA because the proposed method is based on the PC-THz comb induced in PCA. From the previous researches for the active frequency multiplier chain source at 0.3 THz [14], THz-QCL at 2.5 THz [23], and electromagnetic THz comb modes over 0.1~4 THz [11, 17], the measurable range should be extended over several THz, which can cover the frequency bands of high speed wireless communication. The real-time determination of such higher f_{THz} should maintain the same accuracy and precision as that of lower f_{THz} here because they are secured by the frequency standard.

6 Conclusions

To allow commercial femtosecond lasers to be used for the absolute frequency measurement of CW-THz radiation, we demonstrated the real-time, precise measurement of f_{THz} with an f_{rep}-modulated or f_{rep}-free-running PC-THz comb. Based on the fact that an equal interval between PC-THz comb modes is always maintained regardless of the fluctuation in f_{rep}, the PC-THz comb induced by an unstabilized laser was effectively used to determine f_{THz} in fixed-frequency and frequency-fluctuating CW-THz radiation. Using the f_{rep}-free-running PC-THz comb, the absolute frequency f_{THz} of CW-THz radiation was determined with an accuracy of 8.2×10^{-13} and a precision of 8.8×10^{-12} at a measurement rate of 10 Hz. Furthermore, even when f_{THz} suddenly crossed 394 modes in the PC-THz comb due to a large fluctuation over 20 GHz, f_{THz} was correctly determined in real time. Therefore, the proposed method is highly promising not only for frequency-stabilized CW-THz sources but also practical CW-THz sources with free-running operation or mode hopping.

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